IMO 2019

60TH INTERNATIONAL MATHEMATICAL OLYMPIAD

Honorary Patrons: Sir Timothy Gowers FRS, Dame Frances Kirwan FRS, Sir Andrew Wiles FRS

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This problem is a seasonal present from IMO 2019 to all our friends. Please share it.

An *input* is a string of zeros and ones. Write down a row below it using the Pascal triangle rule, where each number is the sum of the two numbers diagonally above it. We are working "modulo 2" so $1+1=0$. Repeat until you form a triangle of numbers. Here is an example, where the input is 1 0 1 1 1.

$$\begin{array}{ccccccccc} 1 & & 0 & & 1 & & 1 & & 1 \\ & 1 & & 1 & & 0 & & 0 & \\ & & 0 & & 1 & & 0 & & \\ & & & 1 & & 1 & & & \\ & & & & 0 & & & & \end{array}$$

The *output* of this procedure is the string read along the side of the triangle from the bottom to the top right, in that order. In this case the output is 0 1 0 0 1.

We decide to use a string of length n as input, so there are 2^n possible input strings. How many of these strings have the property that they are the same as their output string?

No complicated calculations are necessary. Use only beautiful ideas.

Merry Christmas and Happy New Year from the IMO 2019 organizers

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The International Mathematical Olympiad 2019 in the UK

www.imo2019.uk